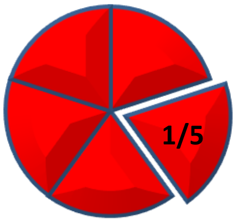
Fraction Facts

Definition: A fraction is a numerical representation for part of a whole.



The **DENOMINATOR** tells how many equal pieces the whole is divided into.

$$\frac{1}{5}$$

The **NUMERATOR** tells how many pieces of the whole the fraction represents.

Add all the pieces to get the whole:

$$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{5}{5}=1$$

Fact: $\frac{x}{x}=1\ (x\neq 0)$

The fraction bar represents **division** (÷), so $\frac{1}{5}=0.2,\ \frac{10}{2}=5,\ and\ \frac{38}{5}=7.6.$

Any fraction with a Denominator of 1 is equal to its Numerator: $\frac{x}{1}=x\div 1=x$

Division by zero is Undefined, so the Denominator of a fraction can never be zero: $\frac{x}{y}\ (y\neq 0)$

Fundamental Property of Fractions . . .

$$\frac{ax}{bx}=\frac{a}{b}$$

*We use this fact when we **Reduce** (or **Simplify**) fractions to lowest terms.

$$\frac{4}{10}\rightarrow\frac{2\bullet 2}{5\bullet 2}\rightarrow\frac{2}{5}\bullet 1=\frac{2}{5}$$

Equality of Fractions . . .

$$\frac{a}{b}=\frac{c}{d}\quad \text{if and only if}\quad ad=bc$$

*We use this fact when we **Cross Multiply** to solve for an unknown numerator or denominator.

$$\frac{x}{10}=\frac{2}{5}\rightarrow 5x=20\rightarrow\frac{5x}{5}=\frac{20}{5}\rightarrow x=4$$

Addition and Subtraction of fractions require a common denominator.

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}\quad and\quad \frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$$

When the denominators are different, multiply one or both fractions by another fraction that is the equivalent of 1 $\left(\frac{x}{x}\right)$ to create a **Common Denominator**; then add or subtract.

$$\frac{a}{b}+\frac{c}{d}=\frac{a}{b}\left(\frac{d}{d}\right)+\frac{c}{d}\left(\frac{b}{b}\right)=\frac{ad+cb}{bd}$$

You may be able to multiply the smaller Denominator by something to create the larger one:

$$\frac{1}{2}+\frac{3}{4}\rightarrow\frac{1}{2}\left(\frac{2}{2}\right)+\frac{3}{4}\rightarrow\frac{2}{4}+\frac{3}{4}=\frac{5}{4}$$

If not, then multiply the two Denominators together:

$$\frac{1}{2}-\frac{5}{7}\rightarrow\frac{1}{2}\left(\frac{7}{7}\right)-\frac{5}{7}\left(\frac{2}{2}\right)\rightarrow\frac{7}{14}-\frac{10}{14}=-\frac{3}{14}$$

Multiplication and Division of fractions <u>do not</u> require a common denominator.

$$\frac{a}{b}\bullet\frac{c}{d}=\frac{ac}{bd}$$

Note that it is easier to reduce before actually multiplying.

$$\frac{1}{2}\bullet\frac{5}{3}\bullet\frac{2}{5}=\frac{1\bullet\cancel{5}\bullet\cancel{2}}{\cancel{2}\bullet 3\bullet\cancel{5}}=\frac{1}{3}$$

To divide fractions, first <u>invert the Divisor</u> (second fraction) to get its **Reciprocal**; then multiply.

$$\frac{a}{b}\div\frac{c}{d}\rightarrow\frac{a}{b}\bullet\frac{d}{c}=\frac{ad}{bc}$$

Factoring before multiplying can help with reducing:

$$\frac{3}{8}\div\frac{1}{2}\rightarrow\frac{3}{8}\bullet\frac{2}{1}\rightarrow\frac{3\bullet 2}{8\bullet 1}=\frac{6}{8}\rightarrow$$

$$*\ But, note\ that\ 2\ is\ a\ factor\ of\ 8, so\ \ldots$$

$$\frac{3\bullet\cancel{2}}{4\bullet\cancel{2}\bullet 1}=\frac{3}{4}$$

A **Proper Fraction** has a numerator that is smaller than its denominator and represents a quantity **less than the whole, or < 1**:

1/5, **2/5**, **3/5**, and **4/5** are proper fractions.

An **Improper Fraction** has a numerator larger than its denominator and represents a quantity **greater than the whole, or > 1:**

6/5, **10/5**, and **27/5** are improper.

Mixed numbers, such as $7\frac{3}{5},\ 23\frac{6}{7},\ and\ 8\frac{110}{241}$, are whole numbers and portions less than 1 (fractions) added together.

$$7\frac{3}{5}\ means\ 7\ and\ \frac{3}{5}, or\ \ 7+\frac{3}{5}$$

It is often useful in doing calculations to convert mixed numbers to improper fractions. To do so, change the whole number to a fraction with the same denominator as the other fraction and add:

$$7=\frac{7}{1}\times\frac{5}{5}=\frac{35}{5};\ then\ \frac{35}{5}+\frac{3}{5}=\frac{38}{5}$$

A quick way: (WHOLE NUMBER x DENOMINATOR + NUMERATOR)/DENOMINATOR: $7\frac{3}{5}=\frac{7\times5+3}{5}=\frac{38}{5}$

To go from **improper fraction to mixed number**, simply divide the **Numerator** by the **Denominator**. The **Remainder over the Divisor** is the fractional portion.

$$\begin{array}{r} 7 \\ 5\,\overline{)38} \\ \underline{35} \\ 3 \end{array} \rightarrow 7\ 3/5$$

Comparing fractions . . .

Obviously **5/8 > 3/8**, but what about **5/8** and **7/12**? Here's how to tell:

Express each fraction with a **Common Denominator**:

$$\frac{5}{8}\left(\frac{3}{3}\right)=\frac{15}{24}\ and\ \frac{7}{12}\left(\frac{2}{2}\right)=\frac{14}{24}\rightarrow\frac{15}{24}>\frac{14}{24}, so\ \frac{5}{8}>\frac{7}{12}$$

Or, express each as a decimal:

$$\frac{5}{8}=0.625\ and\ \frac{7}{12}=0.5833\rightarrow\ 0.625\ >0.5833\ ...$$

Also, test for **Equality of Fractions** $\left(\frac{a}{b}=\frac{c}{d}\Leftrightarrow ad=bc\right)$:

$$\frac{5}{8}?\frac{7}{12}\rightarrow 5\bullet12\ ?\ 8\bullet7\rightarrow 60>56, so\ \ \frac{5}{8}>\frac{7}{12}$$

Eliminating fractions . . .

A fraction multiplied by its **Reciprocal** equals **1**; use this fact to isolate x and solve an equation:

$$\frac{3}{5}x=2\rightarrow\left(\frac{5}{3}\right)\frac{3}{5}x=\left(\frac{5}{3}\right)\frac{2}{1}\rightarrow 1x=\frac{10}{3},$$

$$so\ x=\frac{10}{3}, or\ 3.33\ ...$$

Multiply through by the **Least Common Multiple (LCM)** of the denominators replaces fractions with whole numbers, making an equation easier to work with:

$$\frac{2}{3}x^2+\frac{5}{6}x=4\rightarrow$$

$$\left(\frac{6}{1}\right)\frac{2}{3}x^2+\left(\frac{6}{1}\right)\frac{5}{6}x=\left(\frac{6}{1}\right)\frac{4}{1}\rightarrow 4x^2+5x=24$$

From Decimals to Fractions to Percents . . .

Decimals can be expressed as fractions with a **Denominator that is a Power of 10**. The number of digits behind the decimal tells how many zeros belong in the denominator. Remember to reduce fractions when possible:

$$.5=\frac{5}{10}=\frac{1}{2},\ .25=\frac{25}{100}=\frac{1}{4},\ .225=\frac{225}{1000}=\frac{9}{40},\ and\ 1.0=\frac{1}{1}=1\ (no\ digits\ behind\ the\ decimal,\ so\ 10^0=1)$$

To express a fraction as a percent, first divide the **Numerator** by the **Denominator**; then multiply the resulting decimal number by 100 (or, simply move the decimal two places to the right):

1/2 = .50 = 50%, 1/4 = .25 = 25%, 9/40 = .225 = 22.5%, and 1/1 = 1.00 = 100%