Converting Repeating Decimals to Fractions

The "Multiply by 10^n, Stack - Subtract - Solve - Simplify" Method

Step 1: Let x equal the repeating decimal number. This step creates an equation, ***x = original decimal number***.

Example 1	Example 2
x = 0.55555 . . .	**x = -1.04242424242 . . .**

Step 2: Identify the repeating digit(s) in the decimal number.

The repeating digit is 5.	**The repeating digits are 42.**

Step 3: Multiply this equation by a **power of 10** to move the repeating digits to the **LEFT** of the decimal point. This step creates a second equation, ***10^n = 2nd decimal number***.

To move one decimal place, multiply by 10^1 (10).	**To move three decimal places, multiply by 10^3 (1000).**
10x = 5.55555 . . .	**1000x = -1042.4242424242 . . .**

Step 4: Look at the original decimal number (from step 1). If the repeating digits aren't already immediately to the **RIGHT** of the decimal point, multiply the second equation by another power of 10 to achieve this. This step creates a third equation, ***$10^{n+?}$ = 3rd decimal number***.

The repeating digit in the step 1 equation is already to the right: x = 0.55555 . . .	**A zero is immediately to the right of the decimal point:** **x = -1.04242424242 . . .** **So, multiply by 10^1 (10).** **10x = -10.4242424242 . . .**

Step 5: Stack the 2nd and 1st—or 2nd and 3rd—equations (make sure to **align the decimal points**), and then **Subtract** the left sides of each and the right sides of each. The resulting difference is a new equation. **Subtraction eliminates the repeating digits!**

10x = 5.55555 . . . **− x = 0.55555 . . .** **9x = 5**	**1000x = -1042.4242424242 . . .** **− 10x = - (- 10.4242424242 . . .)** **990x = -1032**	*Subtraction changes the sign of -10.4242 . . . to positive.*

Step 6: Solve the new equation for x. The result will be a **fraction** (or ratio of integers) . . . but you're not quite done.

Divide each side by 9 to isolate x.	$\frac{9x}{9} = \frac{5}{9}$ **x = 5/9**	$\frac{990x}{990} = \frac{-1032}{990}$ **x = -1032/990** *Divide each side by 990 to isolate x.*

Step 7: Simply the fraction if it's not already in lowest terms. Now you're done!

5/9 is already in lowest terms. **0.55555 . . . = 5/9**	**-1032/990 = -516/495** **-1.424242 . . . = -516/495**	*Both the numerator and denominator are divisible by 2.*

Adapted from "Converting Repeating Decimals to Fractions" at Basic-mathematics.com.

See next side of this sheet for more about converting repeating decimals to fractions. →

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Another Way to Convert Repeating Decimals to Fractions

The "Break-Down" & "Re-Build" Method

This method may be useful when the repeating digits start several decimal places behind the decimal point.

Step 1: Break down the repeating decimal by expressing it as a **sum** of fractions, with the repeating portion at the end (use place values to figure this out). **You do this in order to isolate the repeating part.**

Example

1.873535 . . . = 1/1 + 87/100 + 0.003535 . . . *Note that the whole number 1 = 1/1*

Step 2: Focus only on the repeating portion of the sum (ignore the rest). **Let x = the repeating portion**. Multiply this equation by a **power of 10** to move the repeating digits immediately to the **LEFT** of the decimal point (in other words, to eliminate any zeros preceding the repeating digits).

x = 0.003535 . . .

100x = 0.353535 . . .

Step 3: Express this new repeating decimal number as a **sum** in order to **isolate the repeating part** (refer to step 1).

100x = 35/100 + 0.003535 . . .

Step 4: Recall that **x = 0.003535 . . .** , so you can replace this part with x in the equation above.

100x = 35/100 + x

Step 5: Solve for x. This step should yield the repeating portion expressed as a fraction.

100x = 35/100 + x
– x – x
99x = 35/100

Divide each side by 99 (or multiply by 1/99).

99x * 1/99 = 35/100 * 1/99

Now 0.003535 . . . is a fraction!

x = 35/9900

Step 6: In the sum from step 1, replace the repeating decimal portion with its fractional equivalent.

1.873535 . . . = 1/1 + 87/100 + 35/9900

Step 7: Now that each element of the sum is a fraction, it's time to **re-build** by adding the fractions together. Make sure to use a common denominator. Reduce if needed.

1.873535 . . . = 9900/9900 + 8613/9900 + 35/9900 = 18548/9900 = 4637/2475

1.873535 . . . = 4637/2475

Both the numerator and denominator are divisible by 4.

Adapted from *College Algebra*, 2nd ed., by Paul Sisson, page 11.

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