

Strategy for Graphing Rational Functions

We follow these steps to graph the rational function $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials written in descending powers of x and $\frac{P(x)}{Q(x)}$ is in simplest form.

- 1. Check for Symmetry.** If $P(x)$ and $Q(x)$ involve only even powers of x , or if $f(x) = f(-x)$, the graph is symmetric about the y -axis. Check for symmetry about the origin.
- 2. Look for vertical asymptotes.** The real roots of $Q(x) = 0$, if any, determine the vertical asymptotes of the graph.
- 3. Look for the y - and x -intercepts.** Let $x = 0$. The resulting value of y , if any, is the y -intercept of the graph. The real roots of $P(x) = 0$, if any, are the x -intercepts of the graph.
- 4. Look for horizontal asymptotes.**
 - If the degree of $P(x)$ is less than the degree of $Q(x)$, the line $y = 0$ is the horizontal asymptote.
 - If the degrees of $P(x)$ and $Q(x)$ are equal, the $y = \frac{p}{q}$, where p and q are the lead coefficients of $P(x)$ and $Q(x)$, is a horizontal asymptote.
 - If the degree of $P(x)$ is greater than the degree of $Q(x)$, there is no horizontal asymptote.
- 5. Look for slant asymptotes.** If the degree of $P(x)$ is 1 greater than the degree of $Q(x)$, there is a slant asymptote. To find it, divide $P(x)$ by $Q(x)$ and ignore the remainder.

Quoted from: David Gustafson and Peter Frisk, *College Algebra*, (8th ed. Pacific Grove, CA: Brooks/Cole—Thomas Learning, 2004), p. 285.



**See the Rational Functions Graphing
Aid on the next page of this handout.**

To graph a rational function, find the following:

Symmetry: x-axis y-axis origin

X-intercepts _____

Y-intercepts _____

Vertical asymptotes _____

Horizontal asymptotes _____

Slant asymptotes _____

Extra Points _____

