Lamar State College-Orange

Relationship between Logarithmic Functions & Exponential Functions

DEFINITION: A logarithm is the inverse of an exponential function.

EXAMPLE: Given: $f(x) = y = 3^x$

Find: Inverse of $f(x)$ or $f^{-1}(x)$

Solution: $x = 3^y$

How do you solve for y? Use a logarithmic function: $f^{-1}(x) = y = \log_3 x$

Solving logarithmic functions: To solve logarithmic functions, one needs to know the following relationship between exponents and logarithms.

FORMULAS: $a^y = x \iff y = \log_a x$

EXAMPLES: $10^2 = 100 \iff 2 = \log_{10} 100$

The rules for simplifying and manipulating logarithmic expressions somewhat resemble the rules for exponents.

Operation	Laws of Exponents	Laws of Logarithms
Identity	$a^n = a^z$ so $n = z$	$log_b x = log_b y$ so $x = y$
Identity	$x^1 = x$	$log_b b = 1$ (or $log_b b^1 = 1$) so $log_b b^x = x$
Division ↔ Subtraction	$\frac{x^n}{x^z} = x^n - x^z$	$log_b\left(\frac{x}{y}\right) = log_b x - log_b y$
Multiplication ↔ Addition	$x^n * x^z = x^{n+z}$	$log_b(x * y) = log_b x + log_b y$
Power of a Power (proved by **Addition**)	$(x^n)^3 = x^{3*n} = x^{3n}$ **Proof:** $(x^n)^3 = x^n * x^n * x^n$ $= x^{n+n+n}$ $= x^{3n}$	$log_b(y^1)^3 = 3 * log_b y^1 = 3 \log_b y$ **Proof:** $log_b(y^1)^3 = log_{b(}y * y * y)$ $= log_b y + log_b y + log_b y$ $= 3\, log_b y$

The **Change of base formula** is useful because some calculators can only compute logarithms of base 10 (the common logarithm, often written simply as *log*) or base *e* (the natural log, *ln*, for which base *e* is understood).

$$\log_b y = \frac{log_{10} y}{log_{10} b} \left(\text{also } \frac{log\, y}{log\, b}\right) \text{ or } \frac{ln\, y}{ln\, b}$$