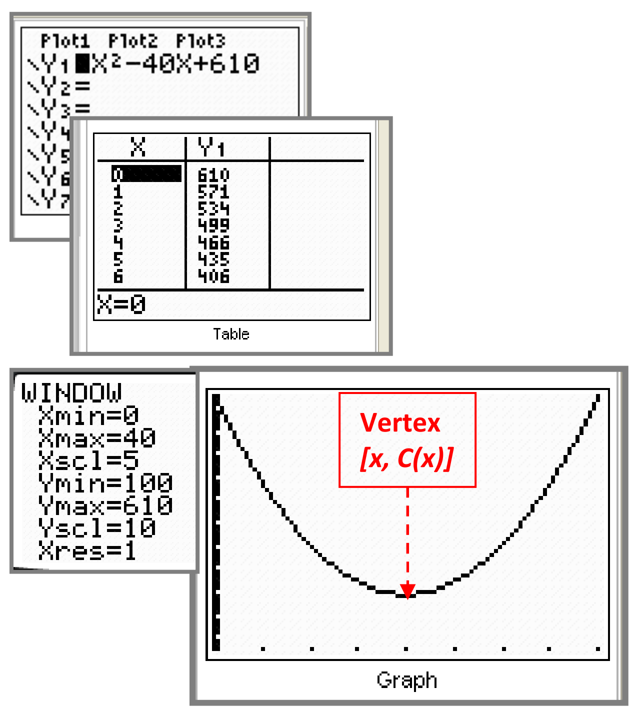
Problem A

Instructions	Example
Step 1: Analyze the problem to determine what information is given and what you are asked to find. ➢ Maxima and minima problems in algebra are solved using ***quadratic equations*** of the form— $y = f(x) = ax^2 + bx + c$ Such an equation may be given, as is the case here, or you may need to use other data given in the problem to create an equation. ➢ In this problem you are asked to find a value of ***x*** (units of tacos) such that ***C(x)*** (the daily operating cost) will be at a minimum, or as low as possible.	Ms. Harris has a taco stand. She has found that her daily costs are approximated by the following equation: $C(x) = x^2 - 40x + 610$ ***C(x)*** is the cost in dollars to sell ***x*** units of tacos. Find the number of units of tacos she must sell to ***minimize*** her cost. Then, find that ***minimum*** cost.
Step 2: Visualize the problem. ➢ Know that the graph of a quadratic equation is always a ***parabola*** (use a graphing calculator to confirm) , and finding maximum or minimum values means identifying the ***x*** and ***y*** coordinates of the parabola's turning point, or ***vertex***. In this problem, the vertex is ***[x, C(x)]***— • ***x*** is the number of units of tacos sold to yield the minimum cost of operation. • ***C (x)*** (which is the same as ***y***) is the minimum cost. ➢ Note that the coefficient of the first term (x^2) is ***positive***, so the parabola is ***concave up*** (hence the vertex is a ***minimum*** point).	The graph of the equation looks like this: 

Step 3: Find the vertex. ➢ Of course the vertex coordinates may be found by examining the graph on a calculator, but perhaps the most efficient method is to use the **vertex formula**— $[-b/2a, f(-b/2a)]$ ➢ ***a*** and ***b*** are the coefficients of the first and middle terms, respectively, in a quadratic of the form— $y = f(x) = ax^2 + bx + c$ ➢ Once you find x ($x = -b/2a$), plug this value into the original equation— $f(-b/2a) = a(-b/2a)^2 + b(-b/2a) + c$ This value is the ***y*** (or ***f(x)***) coordinate of the vertex.	Use the coefficients of the given equation: $C(x) = x^2 - 4\ 0x + 610$ $a = 1$ and $b = -40$ Vertex ***x*** coordinate: $x = -b/2a = -(-40)/2(1) = 40/2 = 20$ $x = 20$ Vertex ***y*** coordinate: $y = C(20) = (20)^2 - 40(20) + 610 =$ $400 - 800 + 610 = -400 + 610 = 210$ $y = C(x) = 210$ The vertex ***[x, C(x)]*** is ***(20, 210)***

Answer to Problem A: Ms. Harris must sell 20 units of tacos in order to achieve a minimum daily operating cost of $210.

Problem B

Instructions	Example
Step 1: Analyze the problem. What do you know? What is being asked? ➢ We know that the problem is asking for a ***maximum***, or the ***largest area***, but no quadratic equation is given. Therefore, we will need to use whatever data and information we have to create an equation. ➢ One piece of numerical data and another piece of information are given— • The total length of fencing is ***4000 meters***. • The ***rectangular*** enclosure will have only ***three sides*** (two of which must be equal). ➢ To find the largest ***area***, we must first	A farmer has ***4000 meters*** of fencing; he wants to enclose a ***rectangular*** garden plot that borders on a river. If the farmer will not fence the side along the river, what is the ***largest area*** he can enclose? What are the dimensions—***length*** and ***width***—of the enclosure?

define ***length*** and ***width*** in order to apply the formula—

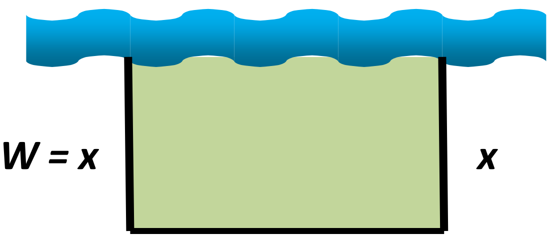
A = L • W

<u>Step 2</u>: Visualize the problem and create the quadratic equation.

- Sketch the proposed enclosure and label.
  - Select a variable (***x***) to represent the unknown width.
  - Using the known total quantity of fencing and the variable, create an expression to define length of the enclosure.
- Plug the ***length*** and ***width*** from the sketch into the area formula (***A = L • W***).
- Express the equation in standard quadratic form—

$$f(x) = ax^2 + bx + c$$

- Note that the coefficient of the first term ($-2x^2$) is ***negative***, so the parabola is ***concave down*** (hence the vertex is a ***maximum point***). You may use a <u>graphing calculator</u> to verify.

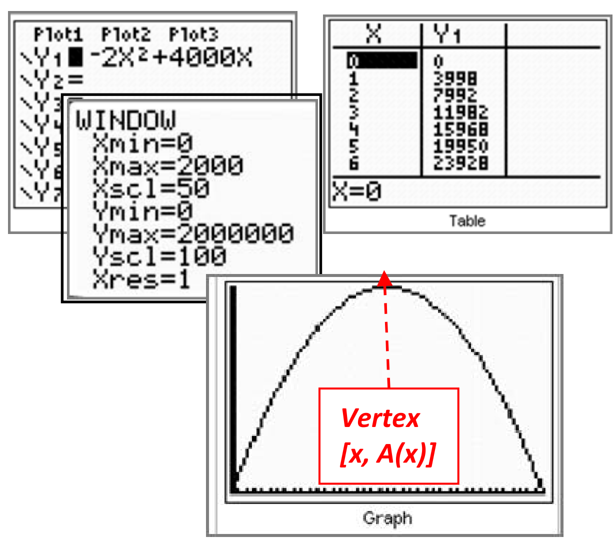


L = 4000 – 2x

$$A = L \cdot W = (4000 - 2x)x =$$

$$4000x - 2x^2$$

$$A(x) = -2x^2 + 4000x + 0$$



<u>Step 3</u>: Find the vertex and use this information to answer all questions.

- Apply the vertex formula—

$$[-b/2a, f(-b/2a)]$$

- ***a*** and ***b*** are the coefficients of the first

Use the coefficients of the equation:

$$A(x) = -2x^2 + 4000x + 0$$

$a = -2$ and $b = 4000$

and middle terms, respectively, in a quadratic of the form— $y = f(x) = ax^2 + bx + c$ ➢ Once you find ***x*** (***x = -b/2a***), plug this value into the original equation— $f(-b/2a) = a(-b/2a)^2 + b(-b/2a) + c$ ➢ The result is the ***y*** (or ***f(x)***) coordinate of the vertex. ➢ Use the values of the vertex coordinates to answer all questions in the original problem.	Vertex ***x*** coordinate: $x = -b/2a = -(4000)/2(-2) =$ $-4000/-4 = 1000$ $x = 1000$ Vertex ***y*** coordinate: $y = A(1000) = -2(1000)^2 + 4000(1000)$ $= -2(1{,}000{,}000) + 4{,}000{,}000$ $= -2{,}000{,}000 + 4{,}000{,}000$ $= 2{,}000{,}000$ $y = A(x) = 2{,}000{,}000$ The vertex ***[x, A(x)]*** is ***(1000, 2,000,000)*** The ***x*** coordinate of the vertex, ***1000***, is the ***length*** of the enclosure, while the ***width*** is ***4000 – 2(1000) = 4000 – 2000 = 2000***. The ***y*** coordinate (***A(x)***)of the vertex, ***2,000,000***, is the ***maximum area*** of the enclosure.

<u>Answer to Problem B</u>: The maximum area the farmer can enclose using 4000 meters of fencing along three sides is 2,000,000 m^2. The dimensions of the enclosure are 2000 meters long by 1000 meters wide.

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<u>More to Consider</u>: What would the maximum area be if the farmer used the 4000 meters of fencing to enclose all four sides? What dimensions would this four-sided enclosure have?

See more quadratic maxima/minima problems at Elizabeth Stapel's Purplemath:
http://www.purplemath.com/modules/quadprob3.html