College Algebra

1. A geometric sequence is a number series in which each successive term results from multiplying or *dividing** the previous term by a constant value called a ***common ratio, r.***

Formula for calculating ***r*** : $r_n = \frac{a_n}{a_{n-1}}$

**n designates the term number (1st, 2nd, 3rd, etc).*

Applying this formula to the given geometric sequence (16, -4, 1, ¼, . . .)

$r_2 = \frac{-4}{16} = -\frac{1}{4},\ r_3 = \frac{1}{-4} = -\frac{1}{4},\ r_4 = \frac{-1/4}{1} = -\frac{1}{4}$

reveals a ***common ratio (r) of -1/4***.

Formula for the n^{th} term of a geometric sequence: $a_n = a_1 r^{n-1}$

To find the 5^{th} term: $a_5 = 16(-1/4)^{5-1} = 16(-1/4)^4 = 16(1/256) = 16/256 =$

1/16

**Without knowing the formulas above, you might also observe that dividing each term by -4 results in the next term*: $\frac{16}{-4} = -4,\ \frac{-4}{-4} = 1,\ \frac{1}{-4} = -\frac{1}{4},\ \frac{-1/4}{-4} = \frac{1}{16}$

Try these sites for more information and practice with geometric sequences:
http://www.mathguide.com/lessons/SequenceGeometric.html

http://www.purplemath.com/modules/series3.htm

http://www.regentsprep.org/Regents/math/algtrig/ATP2/GeoSeq.htm

2. Let ***t*** = *7* days and plug this value into each equation.

$A(7) = 7^2 + 2(7) = 49 + 14 =$ **63**

$B(7) = 10(7) =$ **70 tons**
Maximum Output

3. From the table, ***f(3) = 2***, so substitute ***2*** for ***f(3)*** in ***g(f(3))***.

From the table, ***g(2) =*** **-3** .

x	$f(x)$
-5	7
-2	-5
1	3
3	2

x	$g(x)$
-2	3
1	-1
2	-3
3	-5

f(3)=2

g(2)=-3

4. The ***least common denominator*** of the fractional exponents is ***6***. Multiply by a fraction equivalent to 1 in order to make all denominators the same.

$$X^{1/2(3/3)}\, y^{2/3(2/2)}\, z^{5/6} = x^{3/6}\, y^{4/6}\, z^{5/6} =$$

The ***denominator*** of each fractional exponent is the ***root*** of each variable.
Rewrite the expression using ***radical*** notation:

$$\sqrt[6]{x^3} * \sqrt[6]{y^4} * \sqrt[6]{z^5} = \boxed{\sqrt[6]{x^3\, y^4 z^5}}$$

Try these sites for rules of exponents and more practice with powers and roots:

http://oakroadsystems.com/math/expolaws.htm

http://www.thegreatmartinicompany.com/exponents/exponents-radicals-home.html

http://www.intmath.com/Exponents-radicals/Exponent-radical.php

5. $A - B = \begin{bmatrix} 2 & -4 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} (2-(-2)) & (-4-4) \\ (6-(-6)) & (0-0) \end{bmatrix} = \boxed{\begin{bmatrix} 4 & -8 \\ 12 & 0 \end{bmatrix}}$

Try this site for more information and practice with ***matrices***:

http://www.maths.surrey.ac.uk/explore/emmaspages/option1.html

6. Use these facts to compare possible values: ***f(x) = 2^x, c>1, x>1***

a. If g(x) = cx, then f(g(x)) = 2^{cx} ; cx >1, so 2^{cx} **>2.**

b. If g(x) = c/x, then f(g(x)) = $2^{c/x}$; c/x > 0, so $2^{c/x}$ **> 1.**

c. If g(x) = x/c, then f(g(x)) = $2^{x/c}$; x/c > 0, so $2^{x/c}$ **> 1.**

d. If g(x) = x – c, then f(g(x)) = 2^{x-c} ; even if c is greater than x, making the the exponent "x – c" negative, 2^{x-c} is < 1 but still **> 0.**

e. If g(x) = $\log_c x$, then f(g(x)) = $2^{\log_c x}$. Let $\log_c x$ = y and c^y = x; since X > 1, y (or "$\log_c x$") > 0. *A negative exponent would yield a fraction, and an exponent of 0 would yield 1.* Therefore, $2^{\log_c x}$ **> 0.**

g(x) = cx yields the greatest value for f(g(x)). 2^{cx} will always result in a value greater than 2.

Try this site for more information about functions:

http://www.themathpage.com/aprecalc/functions.htm

Try this site for more information about logarithms:

http://people.hofstra.edu/Stefan_Waner/realworld/calctopic1/logs.html

7. ***f(x+y)=f(x)+f(y)*** holds for all real numbers x and y.

For ***f(0)***, ***x+y=0***; ***x=-y*** and ***y=-x***. Two cases follow from this information.

1) ***x*** and ***y*** are the same number with opposite signs (2 and -2, 5 and -5, etc.)

Substituting **–*x*** for ***y***, ***f(0) = f(x+(-x)) = f(x-x) = f(x) + f(-x)***.

However, possible values for ***f(0)*** cannot be verified before looking at the next case.

2) ***x*** and ***y*** are both 0.

f(0) = f(0+0) = f(0) + f(0) = 2f(0), so

f(0) = 2f(0)

-f(0) -f(0) Subtract f(0) from both sides.

0 = f(0) *OK, f(0) has a value of zero when x and y are zero.*

**Solution above provided by Prof. Elias Jureidini, 8/18/2008.*

Substituting ***x*** for zero (***f(0 + 0) = f(x + x)***) reveals a fact that can be used to prove ***f(0) = 0*** when the variables have the same non-zero value with opposite signs.

f(x + x) = f(x) + f(x) = 2f(x);

f(x + x) can also be expressed as ***f(2x)***, so ***f(2x) = 2f(x).***

Recall ***f(0) = f(x+(-x)) = f(x-x) = f(x) + f(-x).***

If ***f(2x) = 2f(x)***, then ***f(-x) = -f(x)***.

So, ***f(0) = f(x) –f(x) = 0***. *Again, the value of f(0) is zero!*

**Solution above provided by an anonymous tutor at www.mathnerds.com , 8/19/2008.*

8. Powers of ***i*** repeat the following pattern at intervals of 4:

$i^1 = \sqrt{-1} = i$ $\qquad i^5 = i \qquad i^{21} = i$

$i^2 = \sqrt{-1} * \sqrt{-1} = -1$ $\qquad i^6 = -1 \qquad i^{22} = -1$

$i^3 = \sqrt{-1} * \sqrt{-1} * \sqrt{-1}$

$-1 \qquad i = -i$ $\qquad i^7 = -i \qquad i^{23} = -i$

$i^4 = \sqrt{-1} * \sqrt{-1} * \sqrt{-1} * \sqrt{-1}$

$-1 \qquad -1 = 1$ $\qquad i^8 = 1$

Therefore, to find i^{23} divide 23 by 4:

$$\begin{array}{r} 5 \\ 4\,)\,\overline{23} \\ \underline{20} \\ 3 \end{array}$$

A remainder of three means to use first three values in the sequence.

Continued . . .

Determine the sum of one interval: $i + i^2 + i^3 + i^4 = i + (-1) + (-i) + 1 = i - 1 - i + 1 = 0$

That means the sum of the first 4 complete sequences is zero. So, it is necessary only to calculate the sum of the first 3 terms of a sequence:

$$i^{21} + i^{22} + i^{23} = i + (-1) + (-i) = i - 1 - i = \boxed{-1}$$

Try the following site for more help with powers of ***i*** :

http://www.regentsprep.org/Regents/mathb/3c3/powerlesson.htm

9. The formula below is needed to find specific terms in a sequence:

$$a_n = a_1 + (n - 1)d$$

The first term, a_1, is ***3***; however, ***n***, the number of a specific term and ***d***, the common difference between consecutive terms, are unknown.

Two other given values may be used to find ***n*** and ***d*** : a specific term (the last), or a_n, is ***136***, and the sum of the total number of terms is ***1,390***. These values can be plugged into the following formula to find ***n***, number of the last term (***136***):

$$S_n = ½*n(a_1 + a_n)$$

S_n is the sum ***1,390***, a_1 is ***3***, and a_n is ***136***. Plugging in these values will yield ***n***, the number corresponding to the term ***136***.

$$1{,}390 = ½*n(3 + 136)$$

$$2(1{,}390) = [½*n(139)]2$$

$$\frac{2780}{139} = \frac{n(139)}{139}$$

$$\boxed{n = 20}$$

Multiply both sides of the equation by ***2*** to eliminate the fraction (***1/2***).

Divide both sides by ***139*** to find ***n***.

Continued . . .

Now, substitute all known values into the formula for the ***nth*** term to find ***d***.

$$a_{20} = 3 + (20 - 1)d$$

Try these sites for more information about arithmetic sequences.

http://www.mathguide.com/lessons/SequenceArithmetic.html

http://www.purplemath.com/modules/series3.htm

$$136 = 3 + 19d$$
$$-3 \quad -3$$

$$\frac{133}{19} = \frac{19d}{19}$$

$$d = 7$$

Subtract 3 from both sides.

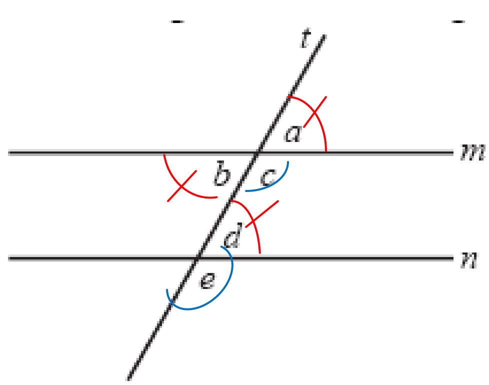
Then divide by 19.

$a_1 = 3$, which was given.
$a_2 = 3 + 7 = 10$
$a_3 = 10 + 7 = 17$

First three terms are ***3***, ***10***, and ***17***.

Geometry

1. Use angle facts to determine which angles are equal.



Try this site for more information about angles and parallel lines:

http://www.ies.co.jp/math/products/geo1/applets/kakuhei/kakuhei.html

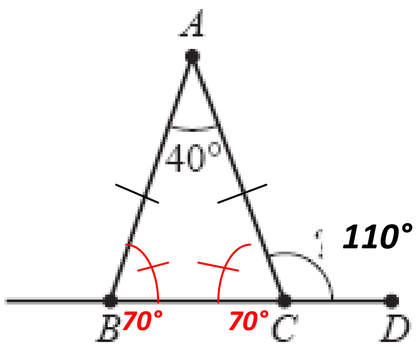
$\angle a = \angle b$ because **vertical angles** created by intersecting lines are equal.

$\angle a = \angle d$ because they are **corresponding angles**, or angles created when a line intersects two parallel lines.

If $\angle a$ equals both $\angle b$ and $\angle d$, then $\angle a = \angle b = \angle d$.

* $\angle c = \angle e$ (corresponding angles), but they do not equal any of the three other angles identified.

2. Use facts about the sum of the angles of a triangle and degree measurement of a straight line.



The sum of all angles of a triangle equals ***180°***. So the sum of the two lower angles of ***Δ ABC*** is ***180° – 40° = 140°***.

Since ***AB*** and ***AC*** are equal, $\angle$***ABC =*** $\angle$***ACB*** and they each measure ***½*** of ***140°***, or ***70°*** each.

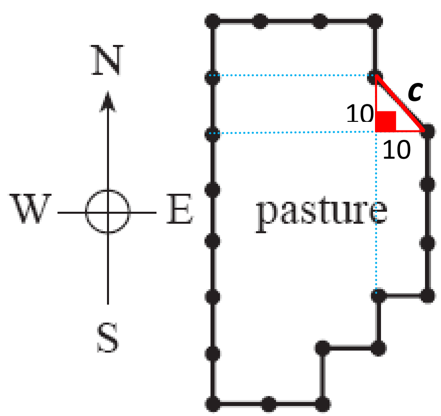
A straight line equals ***180°***, so $\angle$***ACD = 180° – 70° =*** ***110°***.

Try these sites to investigate the sum of angles in a triangle and the sum of angles that form a straight line.

http://argyll.epsb.ca/jreed/math9/strand3/triangle_angle_sum.htm

http://www.walter-fendt.de/m11e/anglesum.htm

3. The perimeter of the pasture is made up of ***twenty 10 ft***. segments (which is the distance between each pair of fence posts), plus one segment that is the ***hypotenuse*** of a right triangle having two legs of ***10 ft.*** each.



P = 20 X 10 + $10\sqrt{2}$ ***= 200 +*** $10\sqrt{2}$.

$\sqrt{2}$ ***is between 1 and 2, so*** $10\sqrt{2}$ ***> 10.***

Therefore, P > 210.

Use the Pythagorean Theorem to find ***c***, the hypotenuse: $a^2 + b^2 = c^2$

$$10^2 + 10^2 = c^2$$

$$100 + 100 = c^2$$

$$200 = c^2$$

$$\sqrt{200} = c$$

$$\sqrt{25 \times 4 \times 2} = c$$

$$(5 \times 2)\sqrt{2} = c$$

$$10\sqrt{2} = c$$

4. Use ***Area = Length X Width*** to find the area of the rectangular garden: ***A = 16 X 9 = 144***.

For a square, all sides are equal, so ***Length = Width***, or $A = s^2$.

Let $s^2 = 144$; therefore, $s = \sqrt{144}$, and $\boxed{s = 12}$

5. Use the ***Pythagorean Theorem*** to solve for the unknown leg length of the right triangle:

$a^2 + b^2 = c^2$ (***a*** and ***b*** are leg lengths, and ***c*** is the hypotenuse)

Let ***a*** = the unknown leg length. Leg ***CB = 3*** and hypotenuse ***AB = 6***.

$$a^2 + 3^2 = 6^2,\ a^2 + 9 = 36,\ a^2 = 36 - 9,\ a^2 = 27,$$

$$a = \sqrt{27} = \sqrt{9 \times 3} = \boxed{3\sqrt{3}}$$

6. Arc length is the length of the curve opposite the central angle. The ratio of the degrees in the central angle (***30°***) to the degrees in the whole circle (***360°***) is proportional to the ratio of the arc length (***6***) to the circle's circumference (***2πr***).

$\frac{central\ angle°}{whole\ circle°} = \frac{arc\ length}{circumference}$, $\frac{30°}{360°} = \frac{6}{2\pi r}$. **Solve for *r* (radius).**

Cross multiply: $\frac{30°}{360°} = \frac{6}{2\pi r}$, $30(2\pi r) = 360(6)$, $60\pi r = 2160$

$r = \frac{2160}{60\pi}$, $\boxed{r = \frac{36}{\pi}}$

Try this site for more information about central angles and arc length:

http://articles.directorym.com/Arc_Length_And_Sectors-a1047348.html

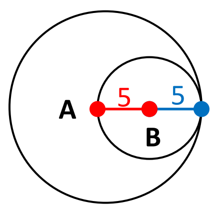
7. Use Volume = Length X Width X Height.

V_1 = 2 X 10 X 6 = 120 sq. in. and ***V_2 = 3 X 5 X h = 15h***.

V_1 = V_2, so ***120 = 15h.***

Solve for h: $\frac{120}{15} = \frac{15h}{15}$, $h = \frac{120}{15}$, ***h = 8 in.***

8. Use $A = \pi r^2$ to find the areas of the larger circle and the smaller circle. The radius of the larger circle is equal to the diameter of the smaller circle: ***2r = 2(5) = 10***.

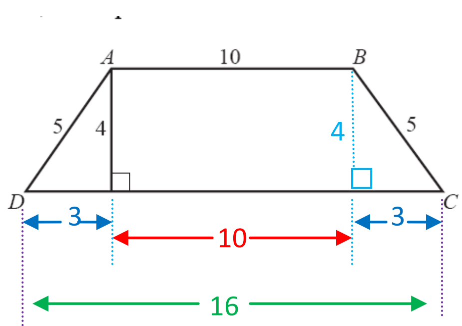


$A\ (larger\ circle) = \pi 10^2 = 100\pi$

$B\ (smaller\ circle) = \pi 5^2 = 25\pi$

Subtract: ***100π − 25π = <u>75π</u>***

9. Find the areas of the middle ***rectangle*** and the ***right triangles*** on each end.



Use the Pythagorean Theorem to find the unknown legs of the right triangles:

$a^2 + 4^2 = 5^2$, $a^2 + 16 = 25$, $a^2 = 9$, ***a = √9 = 3***.

Area of triangle = ½ base*height = $\frac{3*4}{2} = \frac{12}{2} = 6$.

Area of rectangle = ***L X W = 10 X 4 = 40***.

Area of trapezoid: 6 + 40 + 6 = 52

Alternatively, use the formula for the Area of a trapezoid:

A = ($\frac{sum\ of\ two\ bases}{2}$ ***);***

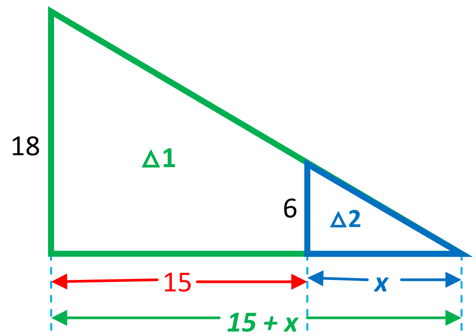
***a** is the altitude, or height.*

$A = 4\left(\frac{10+16}{2}\right) = 4\left(\frac{26}{2}\right)$ ***= 4*13 = 52***

Try this site for more information about area of a trapezoid:

http://www.mathopenref.com/trapezoidarea.html

10. The triangles are similar, so their sides are proportional.



Use the ratios of the known side lengths of each triangle to set up a proportion.

$$\frac{\text{shorter side of } \triangle 1}{\text{longer side of } \triangle 1} = \frac{\text{shorter side of } \triangle 2}{\text{longer side of } \triangle 2}$$

$$\frac{18}{15+x} = \frac{6}{x}$$

Cross multiply and solve for ***x***, which represents the length of the tree's shadow.

18x = 6(15 + x); 18x = 90 + 6x; 12x = 90 ; x = $\frac{90}{12} =$ **7.5**

Or,

$$\frac{\text{shorter side of } \triangle 2}{\text{shorter side of } \triangle 1} = \frac{\text{longer side of } \triangle 2}{\text{longer side of } \triangle 1}$$

$$\frac{6}{18} = \frac{x}{15+x}$$

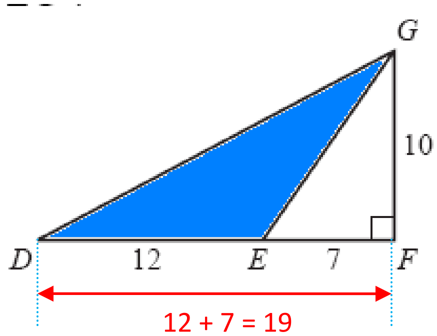
6(15 + x) = 18x; 90 + 6x = 18x; 90 = 12x; x = $\frac{90}{12} =$ **7.5**

Try these sites for information about similar triangles:

http://www.regentsprep.org/Regents/Math/similar/Lstrategy.htm

http://www.mathopenref.com/similartriangles.html

11.



Find areas of larger and smaller right triangles (△DFG and △EFG, respectively) and subtract; the difference is the area of △DEG.

Area of a triangle = ½ base * height = $\frac{b*h}{2}$

The height of both right triangles is 10.

$\Delta DFG = \frac{19*10}{2} = 19 * 5 = 95$

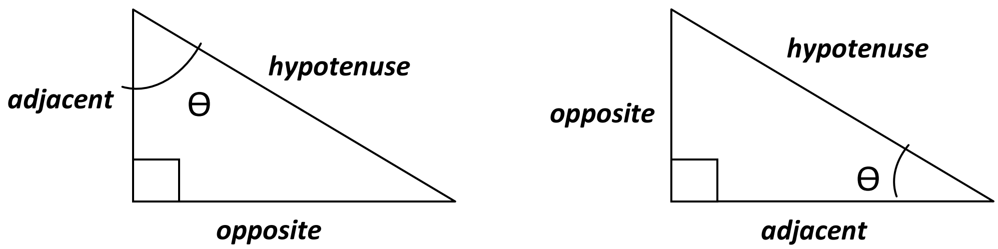
$\Delta EFG = \frac{7*10}{2} = 7 * 5 = 35$

$\Delta DFG - \Delta EFG = \Delta DEG$

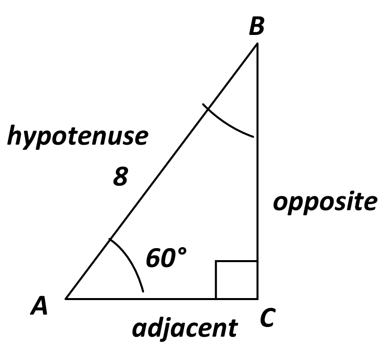
95 – 35 = 60 **Area of** $\Delta DEG = 60$ ***sq. units***

Trigonometry

1. The trigonometric functions relate an angle of a right triangle to the ratio of a pair of the triangle's sides. See the diagrams and chart below. *Θ (theta) is the angle measurement in degrees.



sin Θ	***cos Θ***	***tan Θ***	***csc Θ***	***sec Θ***	***cot Θ***
$\frac{opp.}{hyp.}$	$\frac{adj.}{hyp.}$	$\frac{opp.}{adj.}$	$\frac{hyp.}{opp.}$	$\frac{hyp.}{adj.}$	$\frac{adj.}{opp.}$



Use the fact that ***sin Θ*** = $\frac{opp.}{hyp.}$. $\overline{BC}$ is opposite $\angle$***BAC***, which measures ***60°***. $\overline{AB}$, the hypotenuse, measures ***8*** units; and ***sin 60° = 0.866***. **The given values for **cos 60°** and **tan 60°** are not useful in solving for* $\overline{AB}$.

Let $\overline{BC}$ = ***x*** and substitute known values into ***sin 60°*** = $\frac{opp.}{hyp.}$:

$$0.866 = \frac{x}{8};\ x = 8(0.866);\ x = 6.928 \approx \boxed{6.93}$$

Try these sites for more information on trigonometry and trigonometric functions:

http://www.sparknotes.com/testprep/books/act/chapter10section7.rhtml

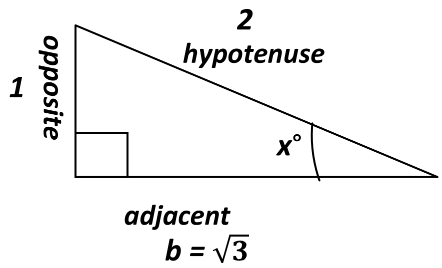
http://www.pballew.net/PCU2.pdf

http://www.sosmath.com/trig/Trig2/trig2/trig2.html

http://math.aa.psu.edu/~mark//Math140/trigident.pdf

2. If **sin α =** $\frac{opp.}{hyp.} = \frac{12}{13}$ and **cos α =** $\frac{adj}{hyp.} = \frac{5}{13}$, then **tan α =** $\frac{opp.}{adj.} = \frac{12}{5}$

3.



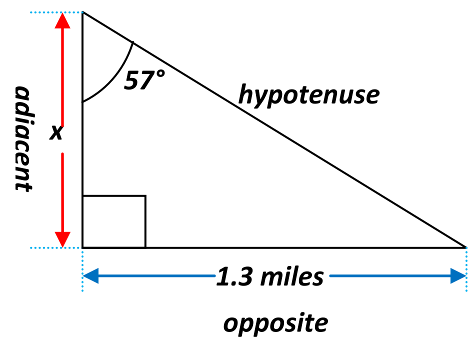
Use ***sin x°*** **=** $\frac{opp.}{hyp.} = \frac{1}{2}$ to identify lengths of two of the a right triangle's three sides.

Then use the Pythagorean formula to find the third side:

$a^2 + b^2 = c^2$; $1^2 + b^2 = 2^2$; ***1 + b² = 4; b² = 3; b*** $= \sqrt{3}$.

$$\cos x° = \frac{adj.}{hyp.} = \frac{\sqrt{3}}{2}$$

4.



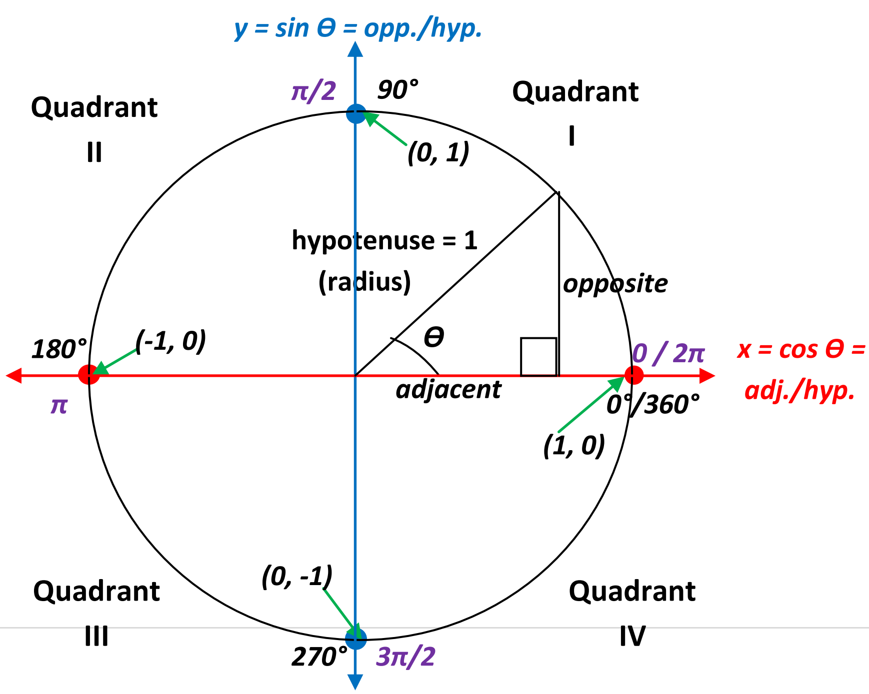
Let x = the height of the balloon from ground.

Use ***tan Θ°*** **=** $\frac{opp.}{adj.}$:

$\tan 57° = \frac{1.3}{x}$; $x(\tan 57°) = x\left(\frac{1.3}{x}\right)$; $\frac{x(\tan 57°)}{\tan 57°} = \frac{1.3}{\tan 57°}$;

$$x = \frac{1.3}{\tan 57°}$$

5.



On the ***unit circle***, the ***y*** coordinate reaches a maximum value of ***1*** at ***Θ = 90°***, or ***π/2 radians***. Therefore, ***y = sin 2x = 1***, and the angle measurement ***2x = π/2*** . Solve for ***x*** by dividing both sides of the equation by ***2***.

$$\frac{2x}{2} = \frac{\frac{\pi}{2}}{2}, \quad x = \frac{\pi}{2} \times \frac{1}{2}$$

*Multiply by reciprocal of divisor.

$$x = \frac{\pi}{4}$$

Try this site for more information about the unit circle:

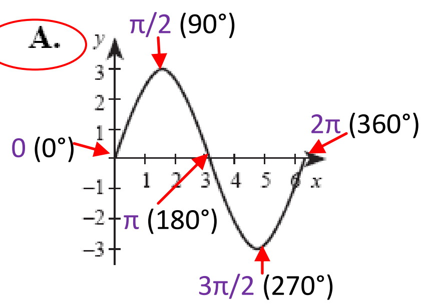
http://www.humboldt.edu/~dlj1/PreCalculus/Images/UnitCircle.html

Try this link for more information about the relationship between degrees and radians:

http://math.rice.edu/~pcmi/sphere/drg_txt.html

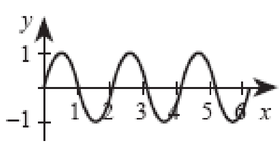
6. Answering this question requires familiarity with graphs of the basic trigonometric functions and their transformations. ***A*** through ***D*** are variations of the ***sine graph***; ***E*** is a ***cosine graph***. Values on the ***y axis*** represent the function outputs; ***0*** through ***6.28*** on the ***x axis*** correspond to ***0*** radians (***0°***) through ***2π*** radians (***360°***). ****2π = 2(3.14) = 6.28***
The function ***y = A sin ϴ*** is not shifted left or right, but its amplitude is greater than one. Only graph ***A*** fits these criteria.

A.



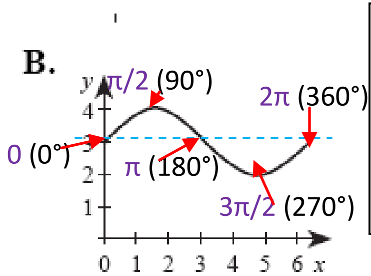
Amplitude ***3***;
Period ***2π***.
y = 3 sin ϴ

C.



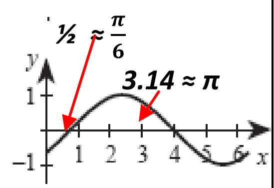
Amplitude ***1***;
Period $\frac{2/\pi}{3}$
(***3*** cycles on graph).
y = sin 3ϴ

B.



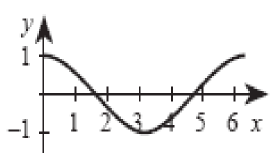
Amplitude ***1***;
Period ***2π***.
Shifted vertically up ***3*** units.
y = sin ϴ + 3

D.



Amplitude ***1***;
Period ***2π***.
Shifted horizontally right about $\frac{\pi}{6}$ ***radians***
y = sin (ϴ − $\frac{\pi}{6}$)

E.



**Not a sine graph.*
Amplitude ***1***; Period ***2π***.
y = cos ϴ

Try this link for graphs of basic trig. functions:

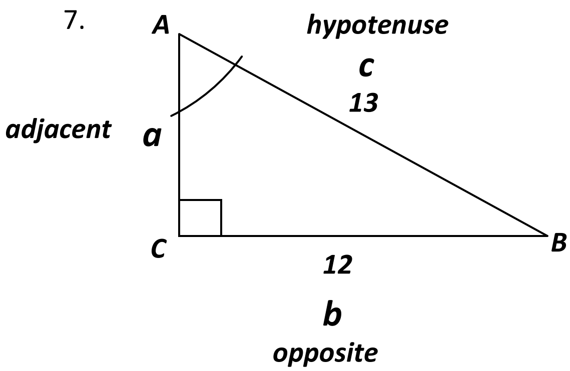
http://www.sparknotes.com/math/trigonometry/graphs/section2.rhtml

Try this link for graphing trig. functions:

http://teachers.henrico.k12.va.us/math/ito_08/08TrigGraphs/8LES2/amp_per_ps_n.pdf

http://colalg.math.csusb.edu/~devel/precalcdemo/circtrig/src/sineshift.html

7.



tan Θ = $\frac{opposite}{adjacent}$; ***tan∠A*** = $\frac{12}{a}$

Find ***a*** ($\overline{AC}$, which is adjacent to ∠***A***) using the Pythagorean theorem:

$$a^2 + b^2 = c^2$$

$$a^2 + 12^2 = 13^2; \; a^2 + 144 = 169$$

Subtract 25 from each side:

$$a^2 = 25$$

Take the square root of each side:

$$a = \sqrt{25}$$

$$a = 5$$

Therefore, ***tan∠A*** = $\boxed{\frac{12}{5}}$